THE AVERAGE TOTAL DAYLIGHT FACTOR

Luisa Brotas\textsuperscript{1,2} Mike Wilson\textsuperscript{1}
\textsuperscript{1}Low Energy Architecture Research Unit, London Metropolitan University, London; United Kingdom
l.brotas@londonmet.ac.uk
\textsuperscript{2}Department of Renewable Energy, National Institute Engineering, Technology and Innovation, I.P., Lisbon, Portugal

Abstract
Daylight calculations are usually based on uniform or overcast skies, but in areas where clear skies are predominant it may underestimate the illuminance levels and might lead to inappropriate urban and window design. In urban canyons reflected sunlight from obstructions and ground is a major contribution to the illumination of buildings in orientations and at times when the sun is behind the building. A simplified calculation that takes into consideration reflected sunlight may be used for quick calculations in the initial design stages of the project. This paper presents the average total daylight factor as a calculation for rooms in urban canyons under sunny climates. It is based on the principle of the integrating sphere and the ratio between the total vertical illuminance on the facade of an urban canyon and global horizontal illuminance. In a similar way to the daylight factor, a method based on the relation between the global horizontal to the total vertical illuminance allows for the sizing of the windows and can be used to evaluate how well a space is daylit under a clear sky.

Keywords: daylight, reflected sunlight, total daylight factor, urban canyon

INTRODUCTION
Daylight calculations are commonly based on the daylight factor regardless of prevailing weather conditions. While this method may be used for overcast sky conditions, it can be argued that is not appropriated for clear skies. In urban canyons reflected sunlight from obstructions and ground can be a major contribution to the illumination of buildings. Simple calculations are most useful in the initial phases of the project and may prevent decisions that compromise the spaces in terms of daylight.

Under sunny conditions, physical measurements collected in an urban canyon in Lisbon showed a linear relationship between the global horizontal illuminance and the total vertical illuminance on the facade when it is not receiving direct sunlight. Further studies carried out with computer simulations with RADIANCE as well as an analytical calculation confirmed this relationship, which is shown to be relatively stable throughout the year, with latitude and orientations and time of day when sunlight is reflected off obstructions and ground. [1]

To a certain extent an urban canyon will tend to behave similarly to a photometric integrator, where the illuminance after interreflections is uniform and independent of the angle of incidence.

This form the basis to the presentation of the average total daylight factor that applies for calculations under sunny skies. It is a simple calculation similar to the average daylight factor but taking into consideration reflected sunlight in an urban canyon. In a similar way to the average daylight factor it may be used as an indicator of how well lit the indoor environment is and allows for the sizing of the windows.

ILLUMINANCE IN THE CANYON

Figure 1. RADIANCE rendering of an urban canyon
Previous research has shown that the ratio of internal to external illuminance may vary greatly under real skies. [2] However, in an urban canyon when direct sunlight is excluded, there is a relationship between the external vertical and the global illuminance.

This relationship can be described by a linear equation of the form:

\[ E_v = k \times E_{gh} + C \]  

(1)

Where \( E_{gh} \) is the global horizontal illuminance and \( E_v \) the total vertical illuminance. The slope \( k \) mainly depends on the reflectance of the obstruction, the geometry of the canyon and the position on the facade. The constant \( C \) is mainly the contribution of the diffuse sky illuminance to the building’s daylight and is more significant at higher floors.

Figure 2. Global horizontal versus north total vertical illuminance for the 21st day of each month

Figure 2 presents a linear relationship between the total vertical illuminance and the global horizontal illuminance for different floor heights in an urban canyon. An approximately linear relationship exists except under specific conditions occurring mainly in the summer when the ground is fully sunlit. However, they do not weight significantly on the average.

The lower floors will tend to have a constant of relatively low value on eq. 1, due to a reduced view of the diffuse sky. If the constant is ignored, there is a direct proportionality between the horizontal and the vertical illuminance. Considering that this relationship does not change significantly during the year, a method based on this relationship could be representative of a year condition, which would be appropriate for window design.

The reflectance of the surfaces (ground and vertical planes) was taken as 0.2. Higher reflectance of surfaces will increase the contribution of reflected light. However, typical building reflectance (where glazing has a reduced diffuse reflectance of around 14%), urban pollution, poor building maintenance, windows reveals and setbacks casting shadows make a conservative value of 0.2 more realistic. This was also corroborated in the real measurements taken in real canyons. Nevertheless, the variation of the reflectance of the surfaces is the factor that affects the most reflected sunlight access to the building in an urban canyon and should therefore be given much thought. The orientation of the building, the latitude of the location and the canyon aspect ratio (between 1:0.5 and 1:1.5) have a lesser effect. [1]

DAYLIGHT FACTOR

The most common method of daylight analysis is the daylight factor approach, where the diffuse internal horizontal illuminance is directly proportional to the diffuse external horizontal illuminance.

However, by definition, sunlight is excluded from the daylight factor calculation, limiting the light source to diffuse from the sky. Besides the simplicity of the calculation it has been widely accepted on the basis of its independence of orientation. Whereas this characterisation is appropriate for heavily cloudy climates, it should not be used in sunny climates. [3]

The average daylight factor, \( \overline{D} \), is the ratio between the mean illuminance in a space and that from an unobstructed external sky, generally assumed to be the CIE overcast sky.

The average daylight factor obtained as an equation and not as an average of daylight measurements on several points in a plane was developed by Lynes. The expression is based on the average illuminance in an enclosure defined by Sumpner. [4, 5] It is defined as

\[ \overline{D} = \frac{\tau A \theta}{2A(1 - \rho_{av})} \]  

(2)
Later work developed by Crisp and Littlefair derived the average daylight factor on the working plane (assumed at 0.85 m). [6]

It is expressed as a percentage as

$$D = \frac{M \tau A_u \theta}{2A(1 - \rho^2\omega)}$$

(3)

Lynes’s formula calculates the average daylight factor over all internal surfaces, whereas the BRE embodies a correction factor to adapt the average illuminance over all surfaces to the horizontal reference plane.

Both eqs. 2 and 3 can be rearranged so as to enable the calculation of the area of the window required to achieve a given average daylight factor. As a rule of thumb, the average daylight factor may be used to characterise the perception of how well a space is lit according to:

- Below 2% the room will appear dull under daylight. Supplementary artificial light will be needed during daylight hours;
- Between 2 and 5% the room will appear increasingly daylit. Electrical task lighting may be needed for visual accuracy;
- Above 5% the room will be strongly daylit. Electrical lighting is rarely needed. However, the excessive dimension of the windows is likely to cause thermal problems.

The average daylight factor will be strongly affected by the reflectance of the interior surfaces. Good reflecting properties will improve the quantity of internal light as well as its distribution, therefore enhancing the quality of the space.

**DAYLIGHT ANALYSIS**

The appearance of a room under daylight not only depends on the illuminance on the working plane, but also on the luminance of the surrounding surfaces and the view through the window. The judgment of the interior brightness will also depend on the occupant’s background and knowledge. People may tend to assume that clear skies provide higher illuminance levels than overcast ones, therefore they are more likely to assume a space is acceptably lit under a blue sky than a cloudy sky. Conversely, a room with a smaller window may appear as dull under an overcast sky as it would under a clear sky, even if the light level may be significantly higher for a sunny sky.

While these criteria can be subjective, as the apparent brightness of the room is strongly dependent on the brightness of the sky and external surfaces seen through the window, the average daylight factor representing the ratio of the indoor to outdoor illuminance is a good indicator of how well daylit a space is. [7] During the day, even if a visual task is performed under task lighting, it is important to have a naturally lit ambient as our circadian rhythm expects a daylit period during the daily routine.

**TOTAL DAYLIGHT FACTOR**

The 'Total Daylight Factor', TD, at a point is the ratio of the total internal illuminance, i.e. direct and indirect for both sky and sun, to the external unobstructed global illuminance.

![Figure 3. Total daylight factor on a working plane (window on top of image) on a 2nd floor room facing north with (left image) and without obstruction (right) in Lisbon in spring equinox. RADIANCE rendering.](image)

Figure 3 represents the total daylight factor on a reference plane in a room facing an obstruction, left image, and without obstruction, on the right. This geometry considers a window area being 10% of the net floor area of the room with double clear glazing.

The room with an obstruction benefits from reflected sunlight deep into in the room resulting in a higher uniformity ratio. The TD is always below 0.5 but this scale has a different order of magnitude to the one indicated for D.

Initial estimations proposed to the average total daylight factors are based on quantitative data obtained with computer simulations. If a daylit appearance is expected, an initial estimation of 0.5% $\overline{TD}$ may be desirable. [1] However, the definition of visual comfort indices should be confirmed with experimental surveys in real situations.

The average total daylight factor (average of values on the left image) is 0.16% which is within a 4% error to the value of 0.17% calculated with the simplified method, to be defined next, for the same geometry.
AVERAGE ILLUMINANCE IN THE ROOM

The principle of the integrating sphere is based on work developed by Sumpner in 1892 with a light source inside a sphere whose inner coating is a perfectly diffusing paint. The luminance of any part of the inner surface, due to light reflected from the rest of the sphere is the same and it is proportional to the total flux emitted by the source. [4, 8]

Similarly, the average illuminance within the room can be based on this principle of interreflection. The analogy has its limitations in the assumption that the light reflected in the canyon is evenly distributed over all the surfaces. In reality, not only are the reflectance and isotropy of various surfaces different, but an urban canyon is rarely free of departures from the ideal geometry. Furthermore, the conversion of a spherical geometry to a parallelepiped one introduces further inaccuracy as the illuminance due to reflected light is not the same everywhere and it is not angularly independent as it is within a sphere. Considering that the canyon is not an enclosed geometry, missing surfaces (side and top) further compromise the uniformity of the space. Nevertheless, the adoption of this simplified model still derives values within an accepted error for simplified calculations.

Let the flux entering the room be $\phi_0$ as

$$\phi_0 = E_v \cdot A_w \cdot \tau$$

(4)

If $A$ is the total area of interior surfaces, ceiling, floor and walls including windows, the average illuminance on the surfaces, $E_v$, due to the flux entering the room is

$$E_v = \frac{\phi_0}{A} = \frac{E_v \cdot A_w \cdot \tau}{A}$$

(5)

The average illuminance due to the first reflected flux, $E_1$ is

$$E_1 = \frac{\phi_1}{A} = \frac{\phi_0 \cdot \rho_{av}}{A} = \frac{E_v \cdot A_w \cdot \tau \cdot \rho_{av}}{A}$$

(6)

The average illuminance due to secondary reflection will be the product of the first reflected flux times the reflectance of the surfaces. And so

$$E_2 = \frac{\phi_1 \cdot \rho_{av}}{A} = \frac{E_v \cdot A_w \cdot \tau \cdot \rho_{av} \cdot \rho_{av}}{A}$$

(7)

The total average illuminance, $E_{in}$, within the room due to multiple reflections is

$$E_{in} = \frac{\phi_0}{A} + \frac{\phi_1 \cdot \rho_{av}}{A} + \frac{\phi_0 \cdot \rho_{av} \cdot \rho_{av}}{A} + ...$$

$$= \phi_0 \left(1 + \rho_{av} + \rho_{av}^2 + ... \right)$$

$$= \frac{\phi_0}{A(1 - \rho_{av})}$$

(8)

Similarly, the average illuminance on the surfaces, $E_v$, due to the flux entering the room is

$$E_v = \frac{\phi_0}{A(1 - \rho_{av})}$$

Then

$$E_{in} = \frac{A_w \cdot \tau}{A(1 - \rho_{av})}$$

(9)

This ratio

$$\frac{E_{in}}{E_v}$$

defines a geometrical relation between the window area and the room area.

The average total daylight factor is

$$TD = \frac{E_{in}}{E_{gh}}$$

(10)

and can be expressed according to eq. 8 as

$$TD = \frac{E_{in}}{E_{gh}} \cdot \frac{A_w \cdot \tau}{A(1 - \rho_{av})}$$

(11)

Previous research has presented a simplified relationship between the vertical and global horizontal illuminance when the building does not receive direct sunlight and is enclosed in an urban canyon. It is defined as eq. 1. Also, it has been proved that this relationship does not change significantly during the year. It may therefore be applied to an equation and thus be representative of a year condition. [1]

Results showed that constant $C$ is mainly the direct contribution from the diffuse sky. Then, if the constant $C$ is excluded, there is a direct proportionality between the horizontal and vertical illuminance, similar to a daylight factor but for clear skies. In those cases the slope
\[ k = \frac{E_{gh}}{E_{av}} \]

can be applied to eq. 11 to obtain the average total daylight factor, expressed as a percentage by

\[ \overline{TD} = \frac{k.A_w.\tau}{A(1 - \rho_{av})} \]  \hspace{1cm} (8)

Where

- \( k \) is the slope of equation previously defined
- \( \tau \) is the diffuse light transmittance of glazing
- \( A_w \) is the net glazed area of window
- \( A \) is the total area of interior surfaces
- \( \rho_{av} \) is the area weighed average reflectance of interior surfaces

**CONCLUSION**

The definition of a simplified calculation for daylight analysis under clear sky distributions is important in order to avoid the use of calculations designed for overcast conditions and their consequent inadequacy.

There is a relationship between the global horizontal illuminance and the vertical illuminance when sunlight is not incident on the facade. This relationship is relatively stable throughout the year.

The slope of the linear relationship is similar for different canyon aspect ratios, but the constant of the equation tends to increase with floor height and for wider canyons due to larger angles of visible sky.

The average total daylight factor calculation is a simple calculation similar to the average daylight factor but taking into consideration reflected sunlight in an urban canyon.

The average total daylight factor may provide a similar characterisation of how well a space is lit.

The average total daylight factor is proportional to the window size, therefore may be a useful method for estimating window sizes in early stages of design. Particularly as it does not require the definition of the window shape or position to be known in advance.

An initial estimation of average total daylight factor as a quarter of the recommended values of the average daylight factor has been put forward. In should be stressed that estimations proposed to characterise a daylit space are based on quantitative data obtained in this study with RADIANCE simulations. The definition of visual comfort indices similar to those assumed for the average daylight factor should mainly be based on experimental surveys in real situations.

**ACKNOWLEDGEMENT**

The first author would like to thank the financial support of INETI – Instituto Nacional de Engenharia e Tecnologia da Informação, I.P. and the FCT – Fundação para a Ciência e Tecnologia on the work developed on her PhD.

**REFERENCES**